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Least-Squares Determination of Burst-Point Coordinates

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This report describes the determination of burst-point coordinates from azimuth and elevation measurements by the method of least squares. The observations are made from a number of observation towers and observed are angles of azimuth and elevation. Both angles are treated as regressand variables, and the burst-point coordinates are determined by minimizing the sum of the correction squares of all observations. Two numerical solution methods are presented, both producing identical results. Methods for the detection of outliers in burst-point observations are shortly discussed. The presented coordinate determination is based on the concept of least-squares regression and differs from another algorithm that has been recently proposed in the Ballistic Research Laboratory Report BRL-MR-3894.

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1. INTRODUCTION

In weapon testing operations, the coordinates of the burst point of an artillery shell are obtained by visual observations using special theodolites that are placed on observation towers (see Roberts 1990 and 1991). The data consist of observed azimuths and elevations of the burst point and the ensuing regression problem is a classic problem of geodesy. Roberts (1991) describes a special treatment of the problem, whereby only the azimuth observations are used to compute the ground coordinates of the burst point. However, the elevation measurements do contribute to the determination of the ground coordinates and should not be ignored. This report describes the treatment of the complete data set in accordance with the least-squares principle whereby the sum of the squared corrections of all angle observations is minimized.

2. BURST-POINT MEASUREMENT PROCESS

To obtain the coordinates of the burst point of an artillery shell, the direction to the burst is measured from a number of observation towers using special theodolites. The observations provide the azimuth and elevation of the burst point. The coordinates of the burst point can be computed from these data if observations from at least two towers are available. In Section 3 we present a least-squares method for this task. The result of the calculations includes estimates of the burst-point coordinates, standard deviations of the coordinates, and correlation coefficients between the coordinates. A typical geometrical arrangement of observation towers and weapon is such that sizable correlations between the coordinate estimates can be expected. Therefore, it is important to have estimates of the correlation coefficients, in particular if an average burst point from various shots is to be calculated, because the variances and covariances of single-shot coordinates enter into the calculation of the average point.

From the coordinates of the burst point, coordinates of the cannon, and the firing direction (defined by an azimuth angle ϕ_w), we compute the *range*, *deflection*, and *firing range* of the cannon. These quantities are defined as follows (see Figure 1):

Range: The *range* r is the distance between the cannon and the projection of the burst point onto the level plane. The projection is called the *impact point.*

Deflection: The *angular deflection* δ is the angle between the firing direction and the direction from the cannon to the impact point. The *metric deflection* d is defined by

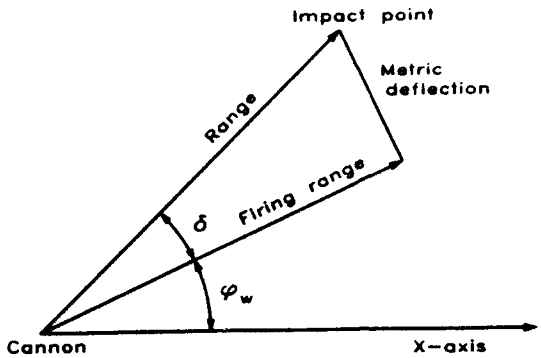

Figure 1. Definition of range and deflection

$$d = r \sin\delta \ . \tag{1}$$

Firing range: The *firing range* f is the component of the distance (range) to the impact point in the firing direction, or

$$f = r \cos\delta \ . \tag{2}$$

3. ESTIMATION OF BURST-POINT COORDINATES

Let the unknown coordinates of the burst point be (x, y, z) and the coordinates of the t-th observation tower be (x_t, y_t, z_t). Let the observed azimuth and elevation be ϕ_t and θ_t, respectively, and let T be the number of observation towers. Then the following relations can be read from Figure 2:

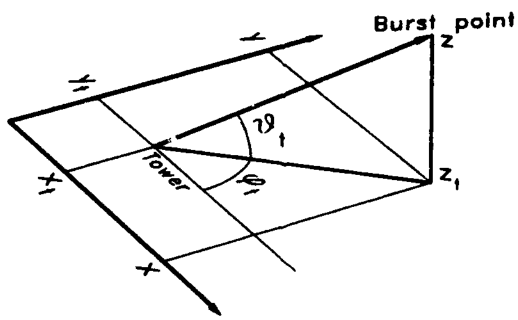

Figure 2. Tower and burst-point coordinates.

$$g_t(\phi_t; x, y) = (x - x_t)\tan\phi_t - (y - y_t) = 0 \ , \qquad t = 1,2,..,T \ , \tag{3}$$

and

$$h_t(\theta_t; x, y, z) = (z - z_t)^2 - [(x - x_t)^2 + (y - y_t)^2]\tan^2\theta_t = 0 \ , \quad t = 1,2,..,T \ . \tag{4}$$

Equations (3) and (4) are the model equations of the regression problem. The regressands are the angles ϕ_t and θ_t, $t = 1,...,T$, the regressor variables are the tower coordinates (x_t, y_t, z_t), and the free model parameters are the burst-point coordinates (x,y,z). Let P_t be estimated variance-covariance matrices of the observations, and let $c_{\phi t}$ and $c_{\theta t}$ be the corrections (residuals) of the observations ϕ_t and θ_t, respectively. Then the least-squares regression problem can be formulated as follows:

$$\text{Minimize} \qquad W = \sum_{t=1}^{T} (c_{\phi t}, c_{\theta t})\, P_t^{-1} \begin{pmatrix} c_{\phi t} \\ c_{\theta t} \end{pmatrix} , \tag{5}$$

$$\left.\begin{array}{ll} \text{subject to} & g_t(\phi_t + c_{\phi t};\, x,y) = 0 \ , \\ \text{and} & h_t(\theta_t + c_{\theta t};\, x,y,z) = 0 \ , \end{array}\right\} \quad t=1,2,\ldots,T \ . \tag{6}$$

This is a least-squares problem with constraints in the form of simultaneous equations. It can be solved, for instance, with the help of the utility routine COLSMU (Celmiņš 1979). However, in the present case, some simplifications apply that allow one to use the simpler routine COLSAC (Celmiņš, l.c.) for least-squares problems with scalar constraints. First, we can assume that all angle observations are independent and have equal accuracies. Therefore, the variance-covariance matrices P_t in Eq. (5) can be set equal to unit matrices and the objective function defined by

$$W = W_1 + W_2 = \sum_{t=1}^{T} c_{\phi t}^2 + \sum_{t=1}^{T} c_{\theta t}^2 \ . \tag{7}$$

Second, because every equation in (6) contains only one scalar observable, the constraints can be defined as the following $2T$ scalar equations:

$$g_t(\phi_t + c_{\phi t};\, x,y) = 0 \ , \qquad t=1,2,\ldots,T \ , \tag{8a}$$

$$h_{t-T}(\theta_{t-T} + c_{\theta,t-T};\, x,y,z) = 0 \ , \quad t=T+1, T+2,\ldots,2T \ . \tag{8b}$$

Eqs. (7), (8a), and (8b) define a least-squares problem with $2T$ observations, $2T$ scalar constraints, and three parameters.

Roberts (1991) suggests for the solution of the regression problem a method that is different from the outlined approach and does not provide the least-squares solution. To show the difference between a least-squares solution and Roberts' method, we first derive a set of normal equations for the least-squares problem. We start by solving the constraint equations (6) for the unknown residuals. The results are

$$\left.\begin{array}{ll} c_{\phi t} = \phi_t - \arctan \dfrac{y - y_t}{x - x_t} \ , & t=1,2,\ldots,T \ , \\[2ex] c_{\theta t} = \theta_t - \arctan \dfrac{z - z_t}{[\,(x - x_t)^2 + (y - y_t)^2\,]^{1/2}} \ , & t=1,2,\ldots,T \ . \end{array}\right\} \tag{9}$$

Substituting these expressions into the objective function (7), we obtain

$$W_1 = \sum_{t=1}^{T} \left(\phi_t - \arctan \frac{y - y_t}{x - x_t} \right)^2 \tag{10}$$

and

$$W_2 = \sum_{t=1}^{T} \left(\theta_t - \arctan \frac{z - z_t}{[\,(x - x_t)^2 + (y - y_t)^2\,]^{1/2}} \right)^2 . \tag{11}$$

Now the only unknowns in the objective function $W = W_1 + W_2$ are the free model parameters x, y, and z. We obtain equations for these parameters by setting equal to zero the partial derivatives of W with respect to the unknowns. The ensuing system of normal equations is

$$\left. \begin{aligned} \frac{\partial W}{\partial x} &= \frac{\partial W_1(x,y)}{\partial x} + \frac{\partial W_2(x,y,z)}{\partial x} = 0 \ , \\ \frac{\partial W}{\partial y} &= \frac{\partial W_1(x,y)}{\partial y} + \frac{\partial W_2(x,y,z)}{\partial y} = 0 \ , \\ \frac{\partial W}{\partial z} &= \frac{\partial W_2(x,y,z)}{\partial z} = 0 \ . \end{aligned} \right\} \tag{12}$$

The least-squares values of x, y, and z are solutions of this system of equations. Numerically solving Eq. (12) is equivalent to solving the least-squares problem (7) and (8) with the utility program COLSAC.

Roberts (1991) solves, instead of the normal equation system (12), the following simpler system:

$$\left. \begin{aligned} \frac{\partial W_1(x,y)}{\partial x} &= 0 \ , \\ \frac{\partial W_1(x,y)}{\partial y} &= 0 \ , \\ \frac{\partial W_2(x,y,z)}{\partial z} &= 0 \ . \end{aligned} \right\} \tag{13}$$

This modification of the normal equation system is justified if one assumes that the dependence of the azimuth angle θ on the altitude z is negligible. The assumption might be true for typical firing tests, but sufficient conditions for the assumption to hold have not been elaborated. If the assumption is not true, then the solution of the modified set, Eq. (13), is different from the least-squares solution. The magnitude of the difference is not known. Hence, because the complete set of normal equations, Eq. (12), provides the least-squares solution under all conditions, we see no reason to use the modified set. Roberts does not give a reason for the neglect of the terms $\partial W_2/\partial x$ and $\partial W_2/\partial y$. Presumably, the terms were removed to facilitate the numerical solution. However, the first two equations of the system (13) constitute a system of simultaneous non-linear equations for x and y that cannot be formally solved. Roberts obtained a numerical solution of that system by linearization and iteration. He could have used the same numerical technique on the complete equation system (12) and obtained the correct least-squares solution.

In the example presented in Section 5, the numerical solution of the modified Eq. (13) is found to be different from the numerical solution of the complete Eq. (12).

A numerical solution of the general regression problem can be obtained also with the help of commercial software for non-linear data fitting. Such a software typically requires that each constraint equation contains exactly one regressand and that the constraints are explicitly solved for the observations; that is, they must be of the type

$$\psi_i = f_i(X_i\,,\beta\,) + c_{\psi i}\ , \tag{14}$$

where ψ_i are the observations (the "regressand variables"), X_i are fixed parameter vectors (the "regressor variables"), β is a free parameter vector of the regression functions f_i, and $c_{\psi i}$ are the corrections (residuals) of the observations. In the present case, this form of constraints can be obtained by solving Eqs. (9) for the observations. (The utility routine COLSAC does not mandate any particular constraint formulation and accepts constraints in the general form $f_i(\psi_i + c_{\psi i}\,, X_i\,, \beta) = 0$. Therefore, we used the simpler implicit formulations (3) and (4) instead of Eqs. (9) in our numerical calculations with COLSAC.) As an example of a commercial software, we chose the program 3R, Release 1990, of BMDP Statistical Software, Inc. To comply with 3R, we expressed the constraints in the form of a regression model as follows:

$$\left.\begin{aligned} &\phi_t\, i_1 + \theta_t\, i_2 = \\ &\quad = \left[\,\arctan\frac{y-y_t}{x-x_t}\,\right] i_1 + \left[\,\arctan\frac{z-z_t}{[\,(x-x_t)^2+(y-y_t)^2\,]^{1/2}}\,\right] i_2 + \\ &\qquad + c_{\phi t}\, i_1 + c_{\theta t}\, i_2\,, \qquad\qquad t = 1, 2, \ldots, T. \end{aligned}\right\} \tag{15}$$

The quantities i_1 and i_2 are called indicator variables. Their values are $i_1 = 1$ and $i_2 = 0$ for azimuth measurements and $i_1 = 0$ and $i_2 = 1$ for elevation measurements. The regressor variables are the tower coordinates x_t, y_t, and z_t, and the components of the free model parameter vector β are the burst-point coordinates x, y, and z.

4. COMPUTATION OF RANGE AND DEFLECTION

Let (x,y,z) be the coordinates of the burst point, (x_w,y_w,z_w) be the coordinates of the gun, ϕ_w be the azimuth angle of the firing direction, and $\vec{n}$ be a unit vector in the direction of firing defined by

$$\vec{n} = \begin{pmatrix} \cos\phi_w \\ \sin\phi_w \end{pmatrix} \quad . \tag{16}$$

We define a range vector by

$$\vec{r} = \begin{pmatrix} x - x_w \\ y - y_w \end{pmatrix} \quad . \tag{17}$$

The range is the length of the range vector, that is,

$$r = \| \vec{r} \| = [\, (x - x_w)^2 + (y - y_w)^2 \,]^{1/2} \quad . \tag{18}$$

The metric deflection is

$$d = [\, \vec{n} \times \vec{r} \,]_z = -(x - x_w)\sin\phi_w + (y - y_w)\cos\phi_w \quad . \tag{19}$$

The angular deflection is

$$\delta = \arcsin(d/r) \tag{20}$$

and the firing range is

$$f = \vec{n} \cdot \vec{r} = r\cos\delta = (x - x_w)\cos\phi_w + (y - y_w)\sin\phi_w \quad . \tag{21}$$

To obtain accuracy estimates of the ranges and deflections, we use the variance propagation formula. Let P_B be the variance-covariance matrix of the burst-point coordinates. It is a 3×3 matrix and one of the outputs of the COLSAC utility routine. Then the variance-covariance matrix of the range, metric deflection, and height of burst is

$$P_{rdz} = \frac{\partial(r,d,z)}{\partial(x,y,z)} P_B \left(\frac{\partial(r,d,z)}{\partial(x,y,z)} \right)^T \quad . \tag{22}$$

Computing the derivatives one obtains the explicit formula

$$P_{rdz} = \begin{pmatrix} (x - x_w)/r & (y - y_w)/r & 0 \\ -\sin\phi_w & \cos\phi_w & 0 \\ 0 & 0 & 1 \end{pmatrix} P_B \begin{pmatrix} (x - x_w)/r & -\sin\phi_w & 0 \\ (y - y_w)/r & \cos\phi_w & 0 \\ 0 & 0 & r^2 \end{pmatrix} \quad . \tag{23}$$

The variance-covariance matrix of the range, angular deflection, and height of burst can be obtained in the same manner and is

$$P_{r\delta z} = \begin{pmatrix} (x-x_w)/r & (y-y_w)/r & 0 \\ -(y-y_w)/r^2 & (x-x_w)/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_B \begin{pmatrix} (x-x_w)/r & -(y-y_w)/r^2 & 0 \\ (y-y_w)/r & (x-x_w)/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{24}$$

The formula for the variance-covariance matrix of the firing range, metric deflection, and height of burst is

$$P_{fdz} = \begin{pmatrix} \cos\phi_w & \sin\phi_w & 0 \\ -\sin\phi_w & \cos\phi_w & 0 \\ 0 & 0 & 1 \end{pmatrix} P_B \begin{pmatrix} \cos\phi_w & -\sin\phi_w & 0 \\ \sin\phi_w & \cos\phi_w & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{25}$$

The formula for the variance-covariance matrix of the firing range, angular deflection, and height of burst is

$$P_{f\delta z} = \begin{pmatrix} \cos\phi_w & \sin\phi_w & 0 \\ -(y-y_w)/r^2 & (x-x_w)/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_B \begin{pmatrix} \cos\phi_w & -(y-y_w)/r^2 & 0 \\ \sin\phi_w & (x-x_w)/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{26}$$

5. EXAMPLES

We use the examples from Roberts (1991). The coordinates of four observation towers and a gun are given in Table 1. (We have subtracted 2 000 m from the x-coordinates and 10 000 m from the y-coordinates for simplicity. Also, the azimuth of the firing direction is modified by subtracting 35° because, according to Roberts (1991), a special coordinate system was used for the measurements.) Table 2 lists observations of azimuths and elevations for eight rounds labeled 11 through 18. The azimuths are again reduced by 35°. Results of regression by the least-squares method (Eqs. (7) and (8), or the equivalent Eqs. (10), (11), and (12)) that were obtained with the utility program COLSAC are shown in Tables 3 and 4. Table 3 contains the estimated burst-point coordinates, translated to a coordinate system with the cannon location (x_w, y_w, z_w) at the origin, and estimates e_x, e_y, and e_z of the standard deviations of the burst-point coordinates. All entries in Table 3 are in metres. Table 4 contains estimated correlation coefficients between the burst-point coordinates.

The variance-covariance matrix P_B of the coordinates is computed from the standard deviations and the correlation coefficients c_{ik} by the formula

$$p_{ik} = e_i \, e_k \, c_{ik} \ , \tag{27}$$

where p_{ik} is an element of the matrix P_B and the indices i and k may take the values x, y, or z.

Identical numerical results were obtained with the utility program 3R, Release 1990, of BMDP Statistical Software, Inc., that uses the constraints (15). This software also provides tests for the dependence of the parameters x, y, and z on data deviations. The tests indicated that the dependence is essentially linear for the ranges of interest. This finding justifies the use of the linearized law of variance propagation for the parameter-variance estimates in the utility routine COLSAC and for the computation of deviations of range and deflection by the formulas of Section 4.

For comparison, Tables 5 and 6 display results obtained by solving the equation system (10), (11), and (13) that correspond to Roberts' solution and agree with the values reported by Roberts (1991). We have supplemented his solution with estimates of coordinate standard deviations and correlation coefficients.

The estimates of coordinates by the two methods differ only by less than 3 m, indicating good quality and consistency of the data. (If the data contain large measurement inaccuracies, then the differences between results from different estimation methods can be significant.) However, the estimated standard deviations vary significantly. For instance, consider the estimates of the standard deviations of angle observations that are listed in Table 7. From the measurement technique, we expect equal uncertainties of azimuth and elevation angle measurements, respectively. Consequently, in our analysis we assumed equal accuracies and the analysis produced for each round one estimate of the standard deviation of all angle measurements. In contrast, Roberts obtained different error estimates for azimuth and elevation readings, respectively, because he used Eq. (13) instead of the least-squares normal equations (12). The consequences are most apparent for Rounds 11, 15, and 16 with larger data scatter. It seems that Roberts' method arbitrarily assigns large errors either to azimuth or to elevation observations making the other observations appear extremely accurate. We fail to see any justification for such an assignment of residuals.

Tables 8 through 11 list the results in terms of firing range, metric deflection, and height of burst. The quantities in the table are given in metres and they were computed from the coordinates of the burst points as described in Section 4. The results are also shown in Figures 3 through 8.

Figures 3, 4, and 5 show the locations of all burst points calculated by the least-squares method. Figures 6, 7 and 8 show the corresponding results obtained by Roberts' method. The accuracies of the burst-point coordinates are indicated by corresponding one-standard-error ellipsoids. We observe in Figure 3 that Rounds 11 and 16 (denoted by "1" and "6", respectively) stand out as less accurate. In contrast, Figure 6 shows that Roberts' method assigns only to Round 11 large errors of ground coordinates, whereas the location error estimates of all other burst points are overly optimistic.

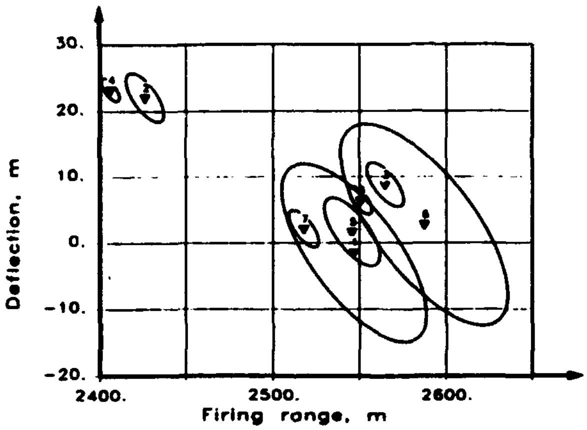

Figure 3. Firing range and metric deflection

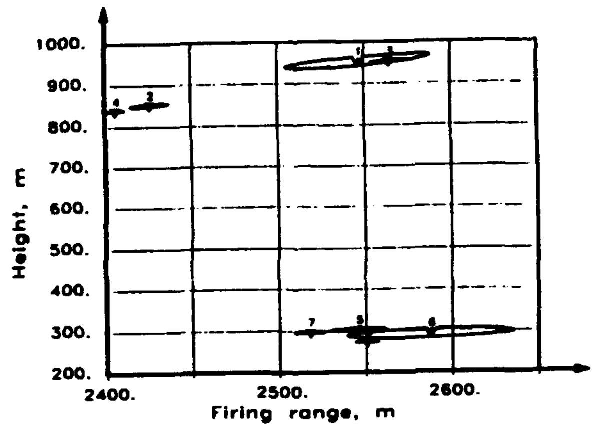

Figure 4. Firing range and height

6. TESTS FOR OUTLIERS

An important part of the treatment of burst-point observations is the detection of gross observational errors. A theoretical basis for the detection with a prescribed confidence level is not available because the problem is not linear and outlier detection theories have been elaborated only for linear regression. One may therefore use statistical tests based on linear regression as approximations or establish ad hoc tests specifically for the problem at hand. We shall discuss both approaches.

Table 12 lists the standard angular deviations e_ψ and the largest residuals of the angle observations for each round (e_ψ denotes the estimate of the standard angular deviation for ϕ and θ, and c_ψ denotes the residual of an angle observation). It is obvious from this list that Rounds 11 and 16 stand out as candidates with bad observations (see also Figure 3).

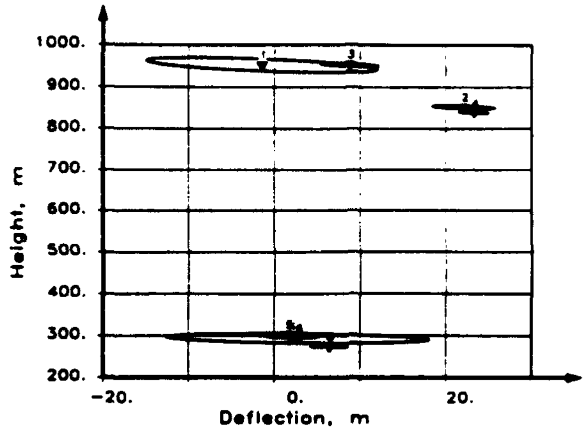

Figure 5. Metric deflection and height

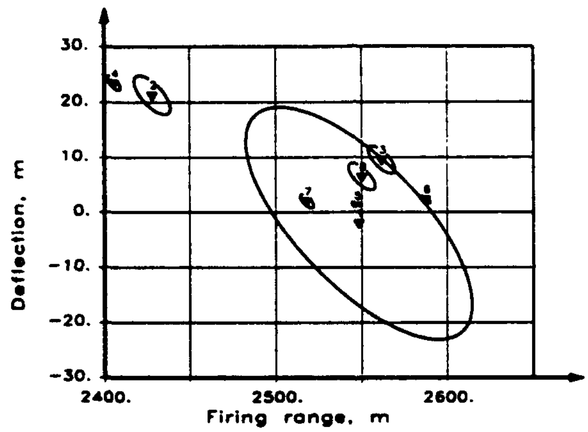

Figure 6. Firing range and metric deflection by Roberts

Another less pronounced outlier might be among the observations of Round 15. A perusal of Table 12 shows that good indicators for rounds with excessive residuals are the magnitudes of e_ψ and $|c_\psi|_{max}$. One can, for instance, postulate for the present arrangement of towers and cannon the following test for a set with outliers:

$$e_\psi > 0.20^\circ \qquad \text{or} \qquad |c_\psi|_{max} > 0.25^\circ \ . \tag{28}$$

For an outlier itself, one might use the same criterion:

$$|c_\psi| > 0.25^\circ \, . \tag{29}$$

To establish tests of this type, one must know what sizes of deviations to expect in a normal operation, that is, the characteristics of the distribution of the observation errors. Thus, the conditions (28) and (29) were derived by comparing results from the eight different

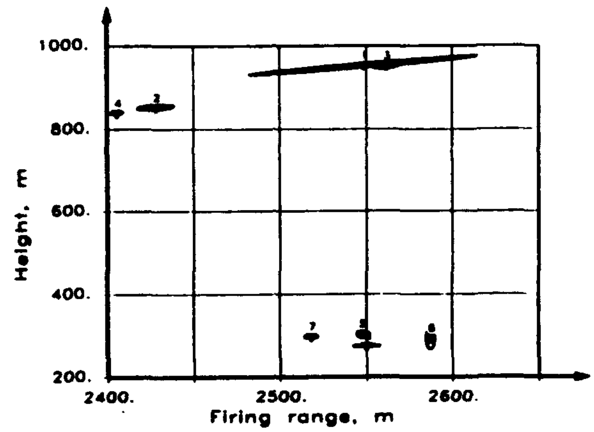

Figure 7. Firing range and height by Roberts

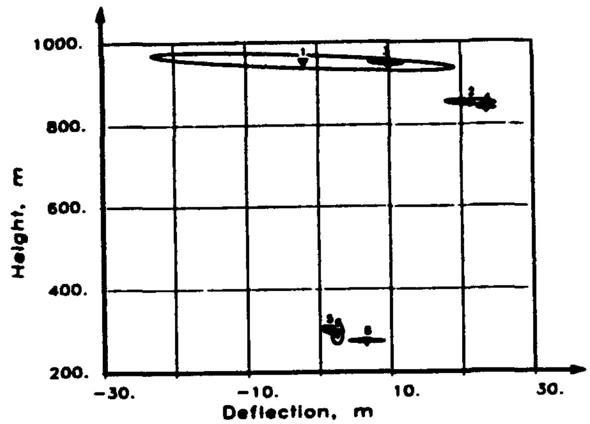

Figure 8. Metric deflection and height by Roberts

rounds in Table 12. Without the possibility to compare between rounds (if only one or two rounds have been measured with the same configuration of towers and cannon) we may use the third column of Table 12 as test statistic. It is obvious that this statistic requires a careful fine tuning of the outlier test, because a large error in one measurement increases the residual $|c_\psi|_{\max}$ as well as e_ψ, and makes their ratio less distinct from those of error-free measurements. After inspecting the data in the third column, one might postulate in the present case the following condition as an error indicator:

$$|c_\psi|_{\max} \,/\, e_\psi > 1.70 \ . \tag{30}$$

Linear regression theory derives a similar test. For a 5% confidence level with eight observations and three parameters, an outlier is indicated if (see Barnett and Lewis 1978, p. 262 ff.)

$$|c_\psi|_{max} \,/\, e_\psi > 2.10 \ . \tag{31}$$

Note that according to this linear theory, all observations would be accepted.

Outlier detection can be also based on repeated regression with one observation deleted in turn (see Barnett and Lewis 1978, p. 244 ff.). This approach is not feasible if the number of observations is large, but with the present eight observations per round, only a moderate amount of additional calculations for the test are needed. The advantage of this test is that it makes a single outlier stand out very pronounced. For illustrative purposes, we first show in Table 13 all relative residuals of the Rounds 11, 15, and 16, that is, of those rounds that are suspect of containing outliers according to the test (30). The observations *Round 11-Tower 3-Azimuth* and *Round 16-Tower 2-Elevation* clearly stand out as suspect. The observation *Round 15-Tower 3-Elevation* is a borderline case and it should be kept in the data set unless there are other indications of errors. We now repeat the regressions, deleting one observation in turn. The idea behind this approach is that the estimated standard deviation is significantly smaller if an outlier is removed from the set but changes only little if a good observation is removed. It can be shown that in linear problems this approach is equivalent to the maximum residual statistics test, Eq. (31), (see Barnett and Lewis 1978, p. 265), but it accentuates the dichotomy between good and bad observations. Table 14 lists the test statistics of the regressions by deleting one observation. Each entry in the table is the ratio of the standard deviation e_ψ of the full set to the standard deviation $\tilde{e}_\psi$ that is obtained if the corresponding observation is removed from the set. Hence, the entries are the relative magnitudes of the standard deviation estimates if the corresponding observations are left in the data set. A large entry indicates a possible outlier. The suspect observations of Round 11 and 16 with standard deviation ratios of 5.3 and 12.1, respectively, stand out more prominently in Table 14 than in Table 13. Outliers might be identified in the present regression problem, for instance, by the ad hoc test

$$\tilde{e}_\psi \,/\, e_\psi > 3. \tag{32}$$

For Round 15, there is no prominent deviation from unity in Table 14. One might check the observation *Round 15-Tower 3-Elevation* for correctness, but one would not discard the measurement based only on the Table 14. We have no such hesitations for the removal of the two outliers for Rounds 11 and 16.

After removing the outliers from the observations of Rounds 11 and 16, we obtain new burst-point coordinates and deviation estimates. The new values are listed in Tables 15 and 16 and shown in Figures 9, 10, and 11. The largest changes are in the deflection of Round 11 that has changed from −1.42 m to +11.39 m and in the height of Round 16 that has decreased by 14.17 m. The overall appearance of the burst-point dispersion has improved as indicated by a comparison of Figure 3 with Figure 9.

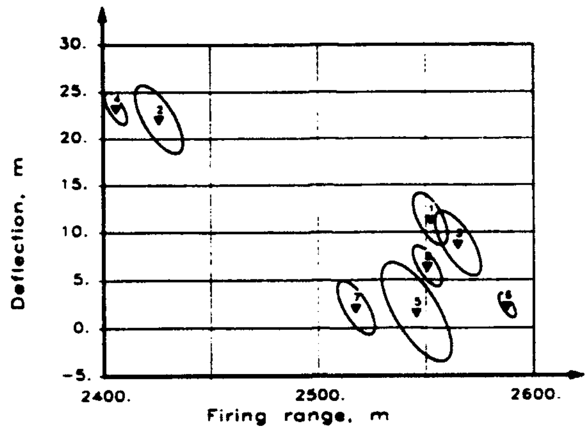

Figure 9. New firing range and metric deflection

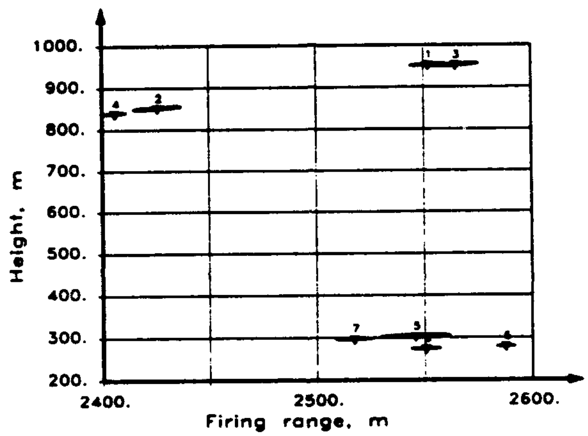

Figure 10. New firing range and height

7. SUMMARY AND CONCLUSIONS

Measurements for the determination of burst-point coordinates consist of azimuth and elevation angles. We compute the coordinates by minimizing the sum of the squares of the residuals of all angle observations. The result is a set of burst-point coordinates that agrees best with the observations in a least-squares sense. In this report, we describe the formalism for the calculation of the coordinates and propose two utility routines for the numerical solution of the regression problem. We also discuss methods for the detection of outliers in the data sets.

The described method for the determination of burst-point coordinates differs from that of Roberts (1991). His method does not minimize the sum of all squared residuals and therefore is not a least-squares algorithm in the usual sense. We show and discuss in an example the differing results that are obtained by the two algorithms. We conclude that

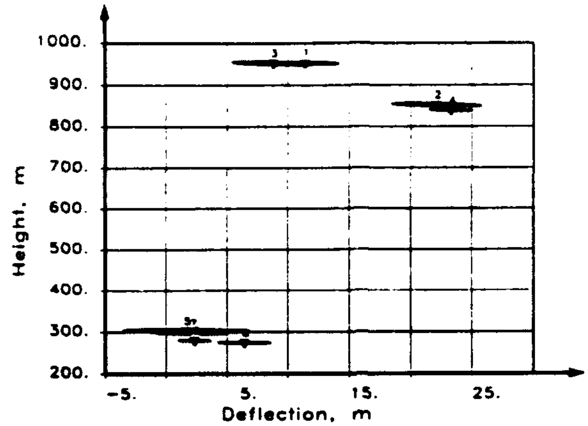

Figure 11. New metric deflection and height

Roberts' algorithm should not be used except when accurate burst-point coordinates are not needed.

In discussing tests for outliers we observe that tests that are based on the absolute size of residuals require the least additional computing. However, they have the drawback that the threshold for outlier detection depends on the setup of the experiments. For instance, a radical change of the firing range or of the arrangement of observation towers would likely require a change of the constants in the tests (28) and (30). A test that is less sensitive to changes in the experiment is based on repeated regressions by deleting one observation in turn. It requires more computation, but the increase of computing time is moderate as long as the number of observation towers is not excessive. We propose to develop a dedicated utility program that includes this test for the computation of burst-point coordinates. Such a program could provide coordinate estimates and outlier tests likely in real time.

Table 1. Tower and cannon coordinates.

	x, m	y, m	z, m
Tower 1	1657.607	2801.626	32.13
Tower 2	1868.530	3870.487	13.30
Tower 3	1754.335	3094.383	4.77
Tower 4	883.790	1998.310	12.13
Cannon	1812.273	2475.803	0.00
Firing direction azimuth $\phi_w = 6° 26'$			

Table 2. Observed azimuths and elevations.

Round	Tower	Azimuth ϕ	Elevation θ
11	1	− 0° 38′	18° 51′
	2	−23° 52′	19° 07′
	3	− 8° 03′	20° 04′
	4	12° 37′	14° 48′
12	1	− 0° 37′	17° 45′
	2	−25° 06′	18° 01′
	3	− 7° 32′	18° 50′
	4	12° 55′	13° 38′
13	1	− 0° 35′	18° 46′
	2	−23° 47′	19° 05′
	3	− 7° 08′	19° 45′
	4	12° 35′	14° 56′
14	1	− 0° 42′	17° 41′
	2	−25° 18′	17° 42′
	3	− 7° 36′	18° 40′
	4	13° 03′	13° 41′
15	1	− 0° 49′	5° 35′
	2	−24° 08′	6° 09′
	3	− 7° 18′	6° 45′
	4	12° 26′	4° 35′
16	1	− 0° 43′	5° 09′
	2	−23° 41′	6° 37′
	3	− 7° 03′	5° 55′
	4	12° 24′	4° 19′
17	1	− 0° 55′	5° 35′
	2	−24° 24′	6° 01′
	3	− 7° 28′	6° 35′
	4	12° 33′	4° 39′
18	1	− 0°43′	5° 08′
	2	−23° 57′	5° 32′
	3	− 7° 15′	5° 50′
	4	12° 35′	4° 17′

Table 3. Estimates of burst-point coordinates.

Round	$x-x_w$	e_x	$y-y_w$	e_y	$z-z_w$	e_z
11	2530.75	42.65	283.92	10.62	952.65	17.43
12	2408.26	11.27	293.79	2.90	852.55	4.44
13	2547.62	10.84	296.12	2.66	952.83	4.41
14	2388.24	5.27	292.72	1.37	839.60	2.07
15	2529.40	16.39	286.87	4.06	303.23	3.90
16	2570.77	49.19	292.70	11.97	293.64	11.48
17	2501.56	8.87	284.22	2.22	297.19	2.12
18	2533.77	6.80	292.17	1.67	274.36	1.58

Table 4. Correlation coefficients to Table 3.

Round	c_{xy}	c_{xz}	c_{yz}
11	−0.45766	0.80939	−0.39097
12	−0.45865	0.78380	−0.38057
13	−0.44965	0.81068	−0.38446
14	−0.45897	0.77999	−0.37980
15	−0.46651	0.42803	−0.21035
16	−0.46006	0.41707	−0.20001
17	−0.46872	0.42178	−0.20888
18	−0.46006	0.39394	−0.19135

Table 5. Roberts' estimates of burst-point coordinates.

Round	$x-x_w$	e_x	$y-y_w$	e_y	$z-z_w$	e_z
11	2532.65	67.28	283.58	16.63	953.28	22.77
12	2410.52	10.83	293.26	2.77	853.26	6.62
13	2544.35	7.86	296.54	1.92	951.75	6.51
14	2387.91	3.97	292.89	1.03	839.50	3.18
15	2530.91	2.70	286.61	0.67	303.38	9.00
16	2570.81	2.59	292.29	0.63	293.65	26.75
17	2501.98	3.93	284.16	0.98	297.24	4.75
18	2533.31	7.90	292.22	1.94	274.32	2.64

Table 6. Correlation coefficients to Table 4.

Round	c_{xy}	c_{xz}	c_{yz}
11	−0.46500	0.97787	−0.47923
12	−0.46531	0.50555	−0.24874
13	−0.45680	0.39813	−0.19151
14	−0.46545	0.38257	−0.18859
15	−0.46741	0.03052	−0.01504
16	−0.46074	0.00942	−0.00455
17	−0.46958	0.08326	−0.04131
18	−0.46079	0.27395	−0.13326

Table 7. Standard deviations of angle observations.

	Least Sq.	Roberts	
Round	e_ϕ and e_θ	e_ϕ	e_θ
11	22.34′	34.76′	5.18′
12	6.37′	6.04′	6.63′
13	5.61′	4.02′	6.54′
14	3.02′	2.52′	3.45′
15	8.48′	1.39′	10.90′
16	24.79′	1.30′	32.00′
17	4.68′	2.07′	5.80′
18	3.51′	4.07′	3.08′

Table 8. Estimates of firing range, metric deflection, and height of burst.

Round	Firing range f	e_f	Deflection d	e_d	Height z	e_z
11	2546.63	41.85	−1.42	13.43	952.65	17.43
12	2426.02	11.05	22.10	3.64	852.55	4.44
13	2564.76	10.64	8.80	3.37	952.83	4.41
14	2406.01	5.17	23.28	1.71	839.60	2.07
15	2545.61	16.08	1.65	5.15	303.23	3.90
16	2587.38	48.28	2.81	15.24	293.64	11.48
17	2517.65	8.70	2.14	2.81	297.19	2.12
18	2550.54	6.67	6.43	2.12	274.36	1.58

Table 9. Correlation coefficients to Table 7.

Round	c_{fd}	c_{fz}	c_{dz}
11	−0.69764	0.80855	−0.59528
12	−0.69136	0.78291	−0.57323
13	−0.69529	0.80987	−0.59373
14	−0.69018	0.77907	−0.57033
15	−0.70427	0.42759	−0.31729
16	−0.70357	0.41672	−0.30701
17	−0.70345	0.42133	−0.31294
18	−0.70133	0.39353	−0.29112

Table 10. Roberts' estimates of firing range, metric deflection, and height of burst.

Round	Firing range f	e_f	Deflection d	e_d	Height z	e_z
11	2548.47	66.01	−1.98	21.12	953.28	22.77
12	2428.20	10.62	21.32	3.49	853.26	6.42
13	2561.55	7.72	9.59	2.44	951.75	6.51
14	2405.69	3.89	23.48	1.29	839.50	3.18
15	2547.08	2.64	1.23	0.85	303.38	9.00
16	2587.37	2.54	2.40	0.80	293.65	26.75
17	2518.06	3.85	2.03	1.24	297.24	4.75
18	2550.10	7.75	6.53	2.47	274.32	2.64

Table 11. Correlation coefficients to Table 10.

Round	c_{fd}	c_{fz}	c_{dz}
11	−0.70343	0.97687	−0.72421
12	−0.69677	0.50499	−0.37219
13	−0.70079	0.39774	−0.29351
14	−0.69525	0.38213	−0.28140
15	−0.70503	0.03049	−0.02266
16	−0.70430	0.00941	−0.00696
17	−0.70413	0.08317	−0.06183
18	−0.70188	0.27367	−0.20259

Table 12. Largest residuals.

Round	e_ψ, degr.	$\lvert c_\psi \rvert_{max}$, degr.	$\lvert c_\psi \rvert_{max}/e_\psi$
11	0.372	0.684	1.836
12	0.106	0.154	1.454
13	0.093	0.136	1.459
14	0.050	0.068	1.351
15	0.141	0.223	1.577
16	0.413	0.786	1.903
17	0.078	0.120	1.540
18	0.058	0.072	1.226

Table 13. Relative residuals $\lvert c_\psi \rvert / e_\psi$.

Round 11. $e_\psi = 0.772°$				
	Tower 1	Tower 2	Tower 3	Tower 4
Azimuth	−0.6996	−0.8267	1.8365	−0.5456
Elevation	0.1848	−0.0391	−0.2917	0.1897
Round 15. $e_\psi = 0.141°$				
	Tower 1	Tower 2	Tower 3	Tower 4
Azimuth	−0.1041	0.0282	−0.0402	0.2228
Elevation	1.2984	−0.3067	−1.5768	0.8191
Round 16. $e_\psi = 0.413°$				
	Tower 1	Tower 2	Tower 3	Tower 4
Azimuth	0.0496	0.0436	−0.0406	0.0317
Elevation	0.7996	−1.9027	0.7433	0.4252

Table 14. Ratios of standard deviations $e_\psi \,/\, \tilde{e}_\psi$.

	Tower 1	Tower 2	Tower 3	Tower 4
Round 11.				
Azimuth	0.971	1.477	5.323	0.957
Elevation	0.899	0.895	0.905	0.898
Round 15.				
Azimuth	0.896	0.895	0.895	0.904
Elevation	1.224	0.906	1.641	0.976
Round 16.				
Azimuth	0.895	0.895	0.895	0.895
Elevation	0.986	12.143	0.974	0.914

Table 15. New estimates of burst-point locations.

Round	Firing range f	e_f	Deflection d	e_d	Height z	e_z	e_ψ
11 old	2546.63	41.85	−1.42	13.43	952.65	17.43	22.34′
11 new	2552.04	7.92	11.39	2.74	953.38	3.29	4.20′
16 old	2587.38	48.28	2.81	15.24	293.64	11.48	24.79′
16 new	2587.60	3.98	2.34	1.26	279.47	1.07	2.04′

Table 16. New correlation coefficients to Table 15.

Round	c_{fd}	c_{fz}	c_{dz}
11	−0.61295	0.80925	−0.53645
16	−0.70370	0.34778	−0.25203

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